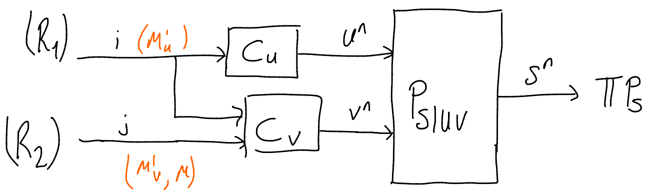
12/06/2016
Tuesday

Superposition SCL

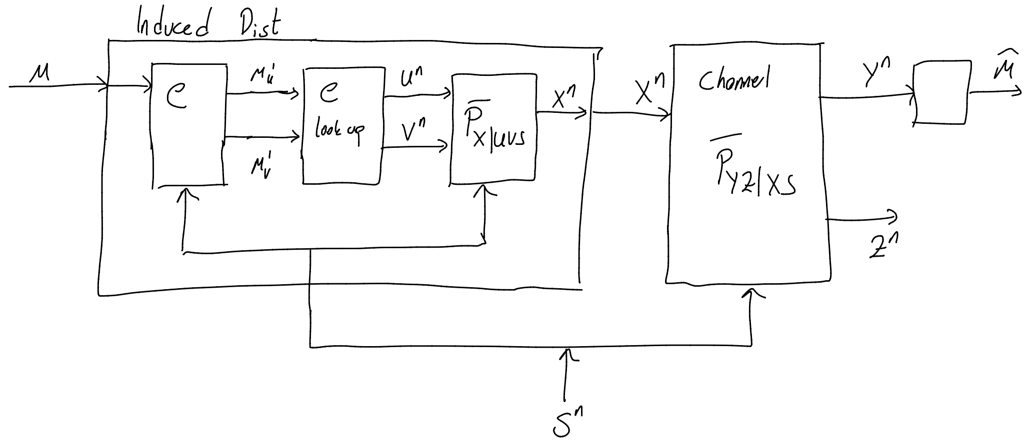


if $R_1 > I(U;S)$

$R_1 + R_2 > I(U,V;S)$

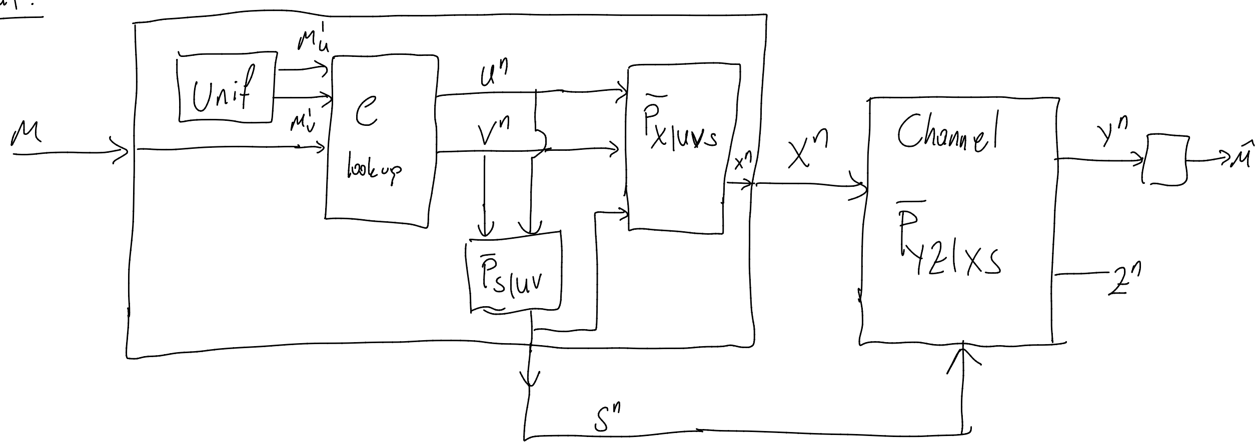
G-P WTC

P



Ideal:

Q



Difference: $P_{S^n M_u' M_v' | M} \neq Q_{S^n M_v' M_u' | M}$

Notice: $P_{S^n|M} = \bar{P}_{S^n}$ by problem definition

$P_{M_u' M_v' | S^n M}$ is anything we want (will be LE)

By Superposition Soft Covering Lemma:

If $R_u' > I(u;s)$

$R_u' + R_v' > I(u,v;S)$ then $Q_{S^n|M} \approx \bar{P}_{S^n}$

see the first picture today!

Define $P_{M_u' M_v' | S^n M} = Q_{M_u' M_v' | S^n M} \propto \bar{P}_{S^n|U^n V^n}(s^n | u^n(m_u'), v^n(m_u', m_v', m))$

C_v has 3 inputs!

This is LE. bayes rule

Therefore given rate constraints $P \approx Q$ (the entire systems!)

$\hookrightarrow$ This is due to SCL.

According to Q:

Secrecy: $\left.\begin{aligned} & R_u' > I(U;Z) \\ & R_u' + R_v' > I(U,V;Z) \end{aligned}\right\} \Rightarrow Q_{Z^n|M} \approx \bar{P}_{Z^n}$

<u>or</u>

for superposition this is better.

$R_v' > I(V;Z|U) \Longrightarrow Q_{Z^n|M} = 2^{-nR_u'} \sum_{m_u'} P_{Z^n|U^n}(z^n | u^n(m_u'))$

Reliability: $\big(\underset{\uparrow R_u'}{u^n(m_u')}, v^n(m_u', \underset{\uparrow R_v'}{m_v'}, \underset{\uparrow R}{m}), Y^n\big) \in T_\varepsilon^{(n)}$

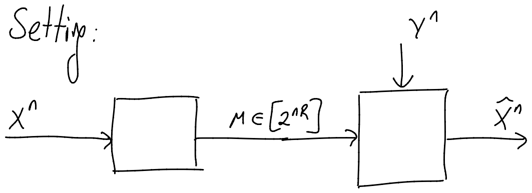
m_u'	m_v' m	distribution	rate bound
wrong	right	$Q_{UV} Q_Y$	~~$R_u' < I(U,V;Y)$~~
right	wrong	$Q_{UV} Q_{Y\|U}$	$R_v' + R < I(V;Y\|U)$
wrong	wrong	$Q_{UV} Q_Y$	$R_u' + R_v' + R < I(U,V;Y)$

doesn't matter because of this!

Wyner - Ziv :

Lossy compression with non-causal side information at decoder.

Setting:



$(X, Y) \sim \bar{P}_{XY}$

Given $d(x, \hat{x})$

D is achievable if

$\mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} d(X_i, \hat{X}_i)\right]$

Equivalently,

$$P_{X^n Y^n M \hat{X}^n} = \bar{P}_{X^n Y^n} P_{M|X^n} P_{\hat{X}|M Y^n} \implies \begin{array}{l} M - X^n - Y^n \\ X^n - (M, Y^n) - \hat{X}^n \end{array}$$

Theorem: (Wyner-Ziv 1976)

$$R(D) = \min_{\substack{P_{U|X} \\ \hat{X} = f(Y, U) \\ \mathbb{E}[d(X, \hat{X})] \leq D}} \underbrace{I(U; X) - I(U; Y)}_{I(U; X|Y)}$$

distribution used in theorem $P_{XYU\hat{X}} = \bar{P}_{XY} P_{U|X} \underset{\substack{\uparrow \\ \text{function of } f}}{P_{\hat{X}|YU}} \implies \begin{array}{l} U - X - Y \\ X - (UY) - \hat{X} \end{array}$

Similar to distribution constraints (on sequence) in problem statement

Main Idea of achievability:

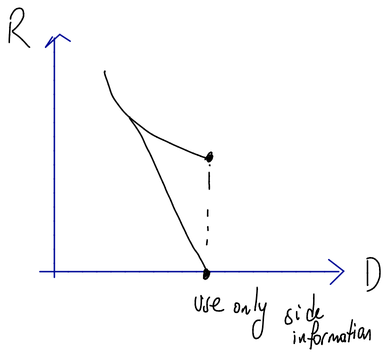
Codebook of U sequences to cover X.

Y boosts the effect in two ways:

1.) Random binning: The codeword can be only partially transmitted. $(-I(U;Y))$

2.) Decoder uses U and Y to produce $\hat{X}$.

In binary symmetric:



Proof of achievability: (Specific)

Fix $\bar{P}$ s.t. $R > I_{\bar{P}}(U;X|Y) = I(U;X) - I(U;Y)$

$D > E_{\bar{P}}(d(X;\bar{X}))$

Choose $R' \in (I(U;X) - R, I(U;Y))$

Let $m' \in [2^{nR'}]$

Codebook $\mathcal{C} = \{u^n(m,m')\} \sim \bar{P}_U$

Encoder: Observe X^n and produce M (somehow)

Decoder: Receive M. Decode $u^n(M, m')$ (somehow)

Compute, for each i, $\hat{X}_i = f(Y_i, U_i)$

given by $\bar{P}$ (i.e. $\bar{P}_{\hat{X}|YU}$)

As long as sequences are $\underbrace{J.T.}_{\text{Jointly typical}} \sim \bar{P}$, distortion satisfied.